

# Mode dispersion in biperiodic photonic crystal waveguides

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(Received 26 August 2003; published 10 December 2003)

We propose and analyze biperiodic photonic crystal waveguides, where there are two sets of discrete translational symmetry along the propagation direction of the photonic crystal waveguides. We present a general method based on a spatial Fourier transform for the calculation of mode dispersion in such biperiodic waveguides. We demonstrate that by modifying the periodicity of only two rows of air holes adjacent to the guiding region, we can achieve guiding through the entire frequency band gap and greatly reduce the group velocity dispersion of the waveguide mode.

DOI: 10.1103/PhysRevB.68.233102

PACS number(s): 42.70.Qs, 42.79.Gn

In recent years, photonic crystals (PCs), which can create a spectral band gap where electromagnetic wave propagation is forbidden in all directions,<sup>1,2</sup> have inspired a lot of interest.<sup>3–6</sup> A complete photonic band gap (PBG) can suppress spontaneous emission, and lead to interesting quantum electrodynamics phenomena.<sup>4</sup> By introducing a line defect, we can create a photonic crystal waveguide (PCW), which can guide light through sharp corners,<sup>3,7</sup> connect PC optical devices of the size of  $\lambda/n$ , and form highly integrated optical circuits.<sup>5</sup> However, due to the distributed Bragg reflection (DBR) along the guiding direction, the fundamental mode in the conventional PCW covers only part of the bandgap and suffers from a large group velocity dispersion (GVD),<sup>6</sup> which may limit its usage in applications such as high-speed optical signal processing in highly integrated optical circuits.

Given the complexity of the conventional photonic crystal waveguide, which is typically defined by a line defect in a two-dimensional array of air holes, it is natural to expect that by modifying the properties of air holes adjacent to the waveguide region, one can significantly improve the transmission and dispersion properties of the PCW. Most of the methods proposed in the literature, which involve modifying the width of the guiding slab or the geometries of the surrounding air holes, can lead to PCWs that support only a fundamental mode,<sup>8,9</sup> but the problems of a limited guiding bandwidth and a large GVD still remain. It was recently proposed to convert a PCW to a normal dielectric slab waveguide with two PC regions surrounding the slab to avoid mode flattening (caused by DBR) inside the PBG.<sup>10</sup> Although the mode dispersion diagram of such a waveguide is linear throughout the PBG, the field extent in the PC region is reduced, which reduces the waveguide-cavity interactions that are usually needed in an integrated PC circuit. Furthermore, the dielectric region in the proposed waveguide is not physically connected and cannot be fabricated in the form of a membrane.

Another method proposed for increasing the guiding bandwidth relies on partially changing the material in the guiding region to another dielectric material with a much smaller refractive index.<sup>11</sup> Although interesting, this method imposes extensive fabrication difficulties; especially in inte-

grated photonic crystal circuits where waveguides with multiple bends are required. Furthermore, the resulting guiding bandwidth (particularly in the more practical versions of the method) is smaller than the full PBG.

In this paper, we propose a method to significantly improve the PCW guiding performance by using a biperiodic PCW, where the periodicity of air holes adjacent to the guiding slab differs from that of the surrounding photonic crystal [Fig. 1(b)]. Such biperiodic PCWs are also theoretically very interesting; since many fundamental results in solid-state physics and PCs (such as the Bloch theorem) no longer apply to biperiodic PCWs. In solid-state physics, we can still describe materials such as amorphous semiconductors using the language of electronic band gap, due to the existence of short-range order. For biperiodic PCWs, however, no such short-range order exists and we can no longer reduce the photonic bands into the first Brillouin zone. In this paper, we present an approach based on spatial Fourier transform (SFT), to analyze the problem of photon guidance in biperiodic PCWs.

We begin by considering a two-dimensional (2D) conventional PCW with a spatial periodicity of  $a$  along the propagation direction. To calculate the mode dispersion diagram of this PCW using the SFT technique, we butt couple the PCW to a slab waveguide, as shown in Fig. 1(a). We can classify the PCW mode according to the Bloch vector  $\beta$  and write the field spatial distribution as  $\vec{u}(x,y)\exp(i\beta x)$ , with  $\vec{u}(x,y) = \vec{u}(x+a,y)$ . Consequently, the SFT spectrum of this field distribution along the  $x$  direction should have peaks located at  $\beta + m(2\pi/a)$  (with  $m$  being an integer), and we notice that the Bloch vector  $\beta$  can be extracted from this SFT analysis without applying the Bloch theorem.

For the biperiodic PCW or a nonperiodic PCW in general, even though the concept of Brillouin zone and the Bloch theorem no longer apply, it is still possible to express the field in such a non-uniform waveguide<sup>12</sup> on the basis of a modal expansion, with  $H_\omega(x,y_0) = \sum_n \hat{H}_{n,\omega} e^{i\beta_n \omega x}$ , where  $\hat{H}_{n,\omega}$  and  $\beta_{n,\omega}$  represent the field component and the propagation constant of the  $n^{\text{th}}$  mode at frequency  $\omega$ , respectively. Therefore, from  $H_\omega(x,y_0)$ , we can extract the peaks of the

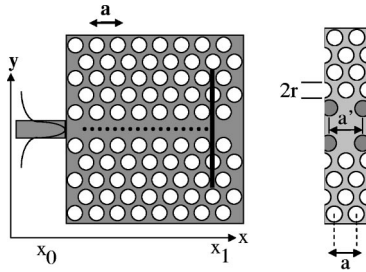


FIG. 1. (a) The basic structure used for the analysis of PCWs with SFT technique. A pulsed Huygens source is placed in the slab waveguide at  $x=x_0$  and the corresponding field is calculated at all points in the same horizontal line along the guiding direction ( $x$ ). For a calculation of the power transmission spectrum, the Poynting vector is calculated and integrated over a surface at  $x=x_1$ . (b) A bi-periodic PCW made by removing one row of air holes from a triangular lattice PC of air holes in Si and introducing a new periodicity in the layers next to the guiding region.

SFT spectrum, which become well-defined quantities and can be used to describe the propagation of PCW modes if the SFT peaks are independent of both the field components and the choice of  $y_0$ .

For nonperiodic waveguides, in general the SFT spectrum contains several peaks, all of which are required to describe the propagation of an optical pulse in the non-periodic waveguides. However, as seen in Fig. 2(a), the SFT spectrum of a bi-periodic PCW contains a single dominant peak, which corresponds to a well-defined propagation constant that describes the waveguide dispersion. Unlike most numerical studies of nonuniform PCWs, which only provide the transmission coefficients,<sup>13</sup> it is possible to use the SFT method to extract the waveguide dispersion that governs the evolution of optical pulses as they propagate in the waveguide.

Hence, we propose a method to obtain the dispersion of a bi-periodic PCW by launching a Huygens source<sup>14</sup> towards the PCW and obtaining the spatial distribution of one of the six electromagnetic field components along the guiding ( $x$ ) direction, with the  $y$  coordinate fixed at  $y_0$  [i.e.,  $H(x, y_0)$ ] as shown in Fig. 1(a). All simulations of this paper are performed using two-dimensional finite difference time domain (FDTD) technique.<sup>15</sup> In our calculations, we use perfectly matched layer (PML) boundary conditions<sup>16</sup> at the four boundaries of the computational domain to simulate the infinite space surrounding the guiding structure and absorb the outgoing radiation.

To establish the validity of the SFT method, we first apply it to analyze the guided mode in a 2D PCW, which is constructed through removing one row of air holes in a triangular lattice of air holes in a silicon substrate. The lattice period and the air hole radius are represented by  $a$  and  $r$ , with  $r=0.3a$ . Since a large PBG can be obtained for transverse magnetic (TM) polarization (magnetic field parallel to the axes of air holes) in these PC structures, we limit our discussions to TM modes. The effective Si dielectric constant is chosen to be  $\epsilon_r=7.9$  to account for the finite thickness of the structure in the normal direction.<sup>17</sup> The PBG for TM polarization in this PC covers the range of  $0.253 < a/\lambda < 0.320$ ,<sup>8</sup> where  $\lambda$  represents the vacuum wavelength. Within the pho-

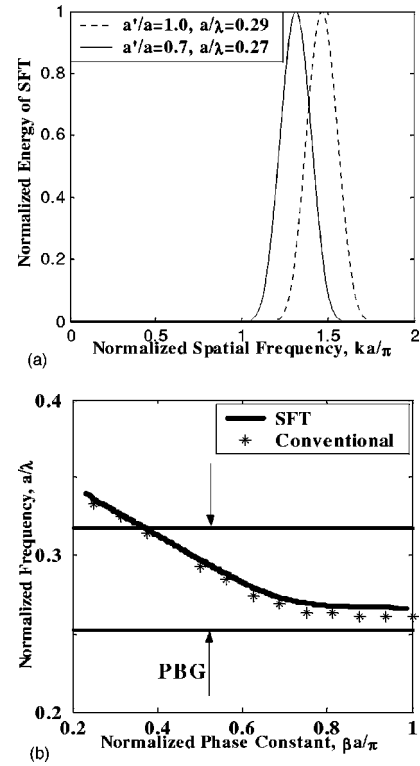


FIG. 2. (a) Normalized energy (magnitude squared) of SFT of the magnetic field for the periodic ( $a'=a$ ) and bi-periodic ( $a'=0.7a$ ) PCWs at different frequencies. Both SFTs have sharp peaks corresponding to well-defined modes. (b) Corresponding dispersion diagram calculated using both the conventional method and the spatial Fourier transform (SFT) technique for the fundamental TM mode for  $r=0.3a$  and  $a'=a$ . Note that due to the periodicity of the PCW, the dispersion diagram has been plotted over the irreducible Brillouin zone by folding all values of the normalized phase constant to the region  $0 \leq \beta a/\pi \leq 1$ .

tonic band gap, the PCW supports two TM modes, one with even symmetry and the other one with odd symmetry. In this paper, we concentrate on the fundamental TM mode, since the odd mode can be pushed out of the PBG by modifying the sizes of the air holes adjacent to the middle slab.<sup>8</sup> We first obtain the dispersion of the PCW by considering a single unit cell of the waveguide [as shown in Fig. 1(b)] and use a conventional order- $N$  spectral approach.<sup>18</sup> We apply the Bloch boundary condition to the left and right side of the unit cell to simulate the infinite waveguide, and use PML boundary condition for the top and bottom of the simulation domain. Then, we obtain the waveguide dispersion through the SFT technique as previously described. The dispersion diagram of the fundamental PCW mode, calculated both by the conventional and the SFT techniques, is shown in Fig. 2(b). The excellent agreement between the two results demonstrates the validity of the SFT technique.

In Fig. 2(a), we show typical SFT spectra of the magnetic field in conventional and bi-periodic PCWs. We notice that, in both cases, most of the field energy is contained within a single spatial Fourier component, which allows us to assign a single propagation constant for the guided bi-periodic waveguide mode. We also repeated the simulations using different

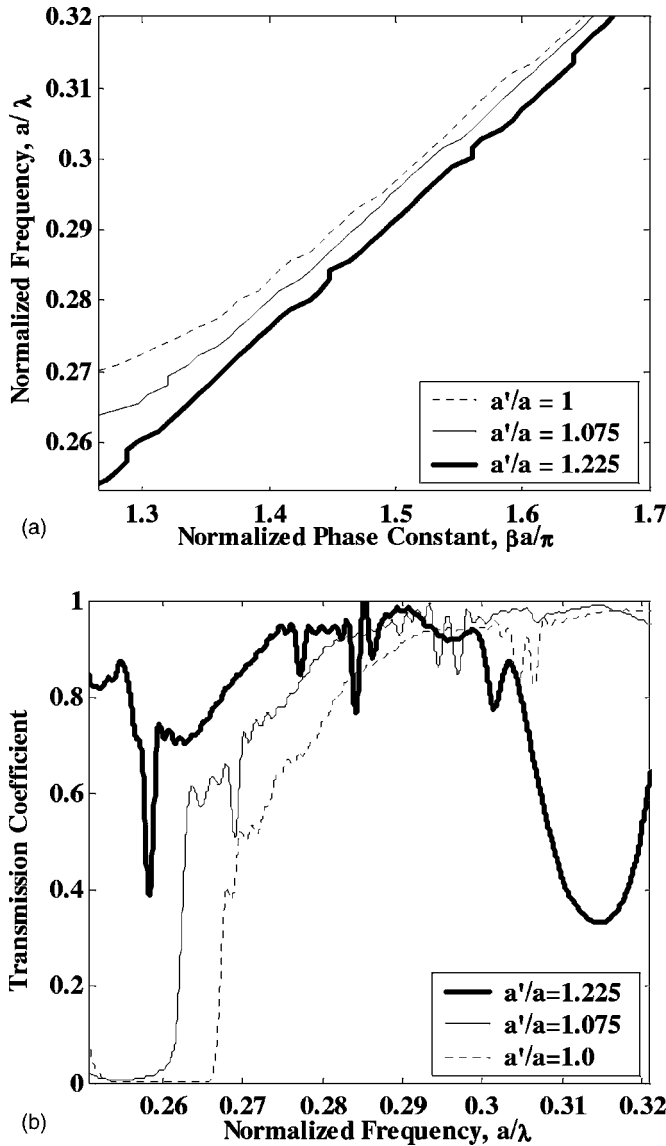


FIG. 3. The evolution of mode flattening and DBR as the periodicity increases: (a) Dispersion diagram of the fundamental TM mode, and (b) power transmission coefficient at  $x_1 = 14a$  for the PCW in Fig. 1 with  $a' \geq a$ . The radius of all air holes is  $r = 0.3a$ .

values of  $y_0$  and different lengths of the biperiodic PCW ( $10a \leq x_1 \leq 18a$ ). The results remain the same.

The finite widths of the peaks in Fig. 2(a) are mainly attributed to the finite number of samples taken in the calculation of the SFT spectra. To verify this, we calculated the Fourier transform of a single complex exponential  $f(x) = e^{j\beta x}$  at  $\beta a/\pi = 1.31$  using the same number of samples and sampling period as in our FDTD and SFT calculations and got the same full width half maximum for the peak as in Fig. 2(a), i.e.,  $\Delta ka/\pi = 0.208$ .

The flattening of the dispersion diagram of the PCW mode in the PBG [see Fig. 2(b)] is very undesirable for optical signal transmission. The primary physical reason for this flattening is the DBR effect, caused by the constructive back-reflection due to the spatial periodicity of the structure in the guiding direction. The simplest demonstration of the

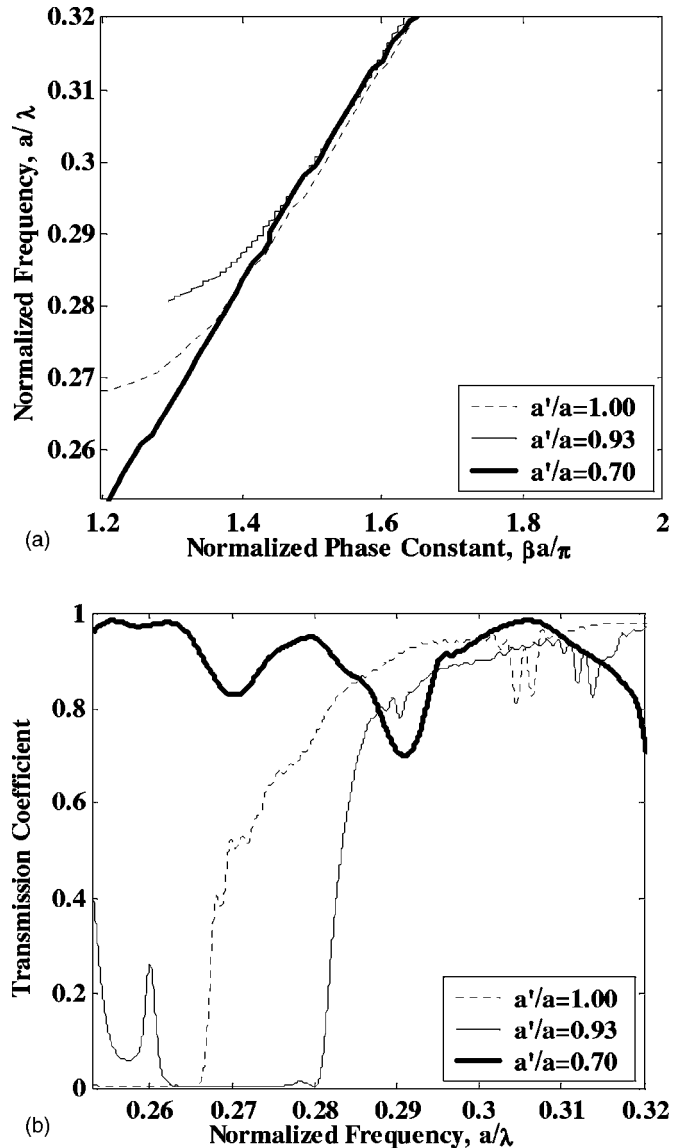


FIG. 4. The evolution of the mode flattening and DBR as the periodicity decreases: (a) Dispersion diagram of the fundamental TM mode, and (b) power transmission coefficient at  $x_1 = 14a$  for the PCW in Fig. 1 with  $a' \leq a$ . The radius of air holes is  $r = 0.3a$ .

DBR effect is the well-known 1D Bragg stack which has been extensively investigated in the literature. In such periodic structures, the DBR wavelength (or frequency) is an increasing (decreasing) function of the period of the structure.<sup>19</sup>

The same effect is responsible for the flattening of the guided mode in the 2D PCW analyzed here [Fig. 1(b)]. To increase the guiding bandwidth in this PCW, we can shift the DBR frequency out of the bandgap by changing the period of the air holes next to the guiding region. Similar to the 1D case, we expect the DBR frequency (wavelength) to increase (decrease) as the new period [ $a'$  in Fig. 1(b)] decreases and decrease (increase) as  $a'$  increases.

Figure 3(a) shows the dispersion diagram of the fundamental TM mode of the PCW shown in Fig. 1(b) for differ-

ent values of  $a' \geq a$ . Note that the concept of irreducible Brillouin zone cannot be applied to the biperiodic waveguide. Thus, the dispersion diagrams shown in Fig. 3(a) are plotted in the *unfolded* form. This explains the existence of  $\beta a/\pi \geq 1$  in Figs. 3(a) and 4(a). As explained earlier, the DBR peak frequency (and thus the flattening frequency) moves to lower frequencies as we increase  $a'$ . In Fig. 3(a), we notice that for  $a' = 1.225a$  the flattening frequency is completely shifted out of the band gap. As the guided mode frequency “enters” into the DBR frequency gap, the modal propagation constant becomes a complex number, whose real part corresponds to the peak of the SFT of the field and the imaginary part accounts for the decaying of the electromagnetic field within the waveguide. The formation of a frequency band gap for the guided PCW mode due to the DBR effect can also be clearly seen from the PCW power transmission spectrum.

To calculate the power transmission spectrum, we use a pulsed Huygens source to excite the fundamental TM mode in the slab waveguide at  $x = x_0$  and integrate the Poynting vector over a surface, which is centered at the middle of the PCW and has a width of 2.5 times the lattice constant (as shown in Fig. 2). The power transmission coefficient was then calculated as the ratio of the transmitted power to the incident power. For more details on the calculation of the power transmission coefficients, the reader should consult Ref. 20.

Figure 3(b) shows the power transmission spectrum calculated at  $x_1 = 14a$  for different values of  $a'/a$  corresponding to the PCWs analyzed in Fig. 3(a). The broad transmission gap in each case is due to the DBR effect. It is clear from Fig. 3(b) that by increasing  $a'/a$ , the DBR peak frequency moves to lower frequencies, as expected. For  $a' = 1.225a$ , the DBR peak (and thus, the mode flattening) frequency is completely out of the PBG and correspondingly the PCW guiding bandwidth covers the entire PBG. We also notice that the flattening frequency for each PCW in Fig. 3(a) coincides with the edge of the transmission minimum in Fig. 3(b).

We can also extend the PCW guiding bandwidth by reducing the period of the holes next to the guiding region

( $a' \leq a$ ), which pushes the DBR frequency out of the band gap. Figures 4(a) and 4(b) show the dispersion diagrams of the fundamental guided TM modes and the power transmission spectra for three PCWs with different values of  $a'/a$ . Figure 4 clearly shows that the mode flattening is shifted to higher frequencies (and eventually out of the PBG) by reducing  $a'/a$ . For  $a' = 0.7a$ , the DBR peak and the transmission minimum are shifted completely out of the PBG, increasing the guiding bandwidth to the entire PBG.

The PCWs presented here allow easy construction of sharp waveguide bends since we have not modified the periodicity of the PCW structure outside the waveguide region (the middle slab and two rows of air holes). Depending on  $a'/a$ , there might be misalignment of only two holes at the corners of the bend. However, it is not a major problem, since the design of efficient bends usually requires the modification of the holes at the bend to increase bending efficiency.<sup>21</sup> Combining the results of this paper with those of Ref. 8, where we demonstrated that single mode guiding in the PCW by changing the sizes of the air holes next to the guiding region, we conclude that the dispersion diagram of a PCW mode can be controlled and designed primarily by the geometry (sizes and periodicities) of air holes next to the middle slab (or guiding region). Modifying the permittivity of the material inside these holes (for example by filling them with an electro-optic polymer and applying an electric field) is another degree of control on the properties of these waveguides. Finally, it is interesting to note that the PCW with  $a' \ll a$ , becomes similar to the structure already proposed in Ref. 10 in the form of a conventional slab waveguide surrounded by two PC regions on both sides.

In conclusion, we demonstrated that a bi-periodic photonic crystal waveguide, where the spatial periodicity of the air holes surrounding the guiding region is modified, can significantly improve the waveguide transmission bandwidth and its group velocity dispersion. We developed a SFT technique that gives the waveguide dispersion without relying on the Bloch theorem, which can also find applications in the analysis of other non-periodic photonic crystal waveguides.

This work was funded by Air Force Office of Scientific Research (G. Pomrenke).

- <sup>1</sup>E. Yablonovitch, Phys. Rev. Lett. **58**, 2059 (1987).
- <sup>2</sup>S. John, Phys. Rev. Lett. **58**, 2486 (1987).
- <sup>3</sup>S. Y. Lin *et al.*, Science **282**, 274 (1998).
- <sup>4</sup>S. Fan *et al.*, Phys. Rev. Lett. **78**, 3294 (1997).
- <sup>5</sup>A. Chutinan, S. John, and O. Toader, Phys. Rev. Lett. **90**, 123901 (2003).
- <sup>6</sup>M. Notomi *et al.*, Phys. Rev. Lett. **87**, 253902 (2001).
- <sup>7</sup>T. Baba, N. Fukaya, and J. Yonekura, Electron. Lett. **35**, 654 (1999).
- <sup>8</sup>A. Adibi *et al.*, Electron. Lett. **16**, 1376 (2000).
- <sup>9</sup>M. Notomi *et al.*, Electron. Lett. **37**, 293 (2001).
- <sup>10</sup>W. T. Lau and S. Fan, Appl. Phys. Lett. **81**, 3915 (2002).
- <sup>11</sup>T. Sondergaard and A. Lavrinenko, Opt. Commun. **203**, 263 (2002).
- <sup>12</sup>B. Z. Katsenelenbaum *et al.*, *Theory of Nonuniform Waveguides: The Cross-Section Method* (Institute of Electrical Engineers, London, 1998).
- <sup>13</sup>C. Jin, B. Cheng, B. Man, Z. Li, D. Zhang, S. Ban, and B. Sun, Appl. Phys. Lett. **75**, 1848 (1999).
- <sup>14</sup>D. E. Merewether, R. Fisher, and F. W. Smith, IEEE Trans. Nucl. Sci. **NS-27**, 1829 (1980).
- <sup>15</sup>K. S. Yee, IEEE Trans. Antennas Propag. **AP-14**, 302 (1966).
- <sup>16</sup>J. P. Berenger, J. Comput. Phys. **114**, 185 (1994).
- <sup>17</sup>L. A. Coldren and S. W. Corzine, *Diode Lasers and Photonic Integrated Circuits* (Wiley, New York, 1995).
- <sup>18</sup>C. T. Chan, Q. L. Yu, and K. M. Ho, Phys. Rev. B **51**, 16 635 (1995).
- <sup>19</sup>A. Yariv and P. Yeh, *Optical Waves in Crystals* (Wiley, New York, 1984).
- <sup>20</sup>A. Adibi *et al.*, Phys. Rev. B **64**, 033308 (2001).
- <sup>21</sup>S. Boscolo, M. Midrio, and T. F. Krauss, Opt. Lett. **27**, 1001 (2002).